SUSY Magnetic Moments Sum Rules and Supersymmetry Breaking

A.Culatti[1]

Dipartimento di Fisica Galileo Galilei
Università di Padova
Via Marzolo 8, 35131 Padova, Italy
INFN, sezione di Padova, Italy

(revised version)

Abstract

It was recently shown that unbroken N=1 Susy relates, in a model independent way, the magnetic transitions between states of different spin within a given charged massive supermultiplet. We verify explicitly these sum rules for a vector multiplet in the case of massless and massive fermions. The purpose of this analysis is to provide the ground for the broken susy case. We study the modifications of these results when an explicit soft Susy breaking realized through a universal mass for all scalars is present. As a by-product we provide a computation of the $g-2$ of the W boson in the standard model which corrects previous evaluations in the literature.

DFPD/94/TH/25 june 1994

[1] e-mail address: culatti@padova.infn.it or culatti@mvxpd5.pd.infn.it
Phone number 0039 (49) 831739 Telefax number 0039 (49) 844245

1 Introduction

It was recently observed that in a $N = 1$ supersymmetric (Susy) invariant theory, the anomalous magnetic transitions among members of a vector or higher spin supermultiplet are related by model independent sum rules [1] . When we consider a vector supermultiplet these rules are very simple. Indeed, calling h the $g-2$ value of the charged vector bosons $W^{\pm}$, it turns out that the anomalous magnetic moments of the fermionic partners of W (the charginos) are equal to $2h$ and the anomalous magnetic transition between W and its scalar partner (the charged higgs $H^{\pm}$) is again equal to h.

The relevant question that we wish to address in this paper is the impact of the breaking of supersymmetry on these sum rules. In other words, one can try to use the abovementioned anomalous magnetic moments sum rules as an indicator of the amount of susy breaking which is present.

In order to perform this analysis we first make an explicit derivation of the sum rule in the minimal supersymmetric standard model (MSSM) in the situation of unbroken Susy. This derivation accomplishes a twofold purpose: first it prepares the ground for an analysis of the departure from the exact sum rule when Susy is broken in different ways, and, then, it allows for a quick reappraisal of the results concerning the anomalous magnetic moment of the W boson in the standard model (SM). We will show that this reanalysis leads to a sizeable correction of the results previously reported in the literature for this computation.

Unless otherwise specified, our notations and conventions are as in [2] where an exactly supersymmetric version of the Weinberg-Salaam $SU(2)_L \times U(1)_Y$ SM is illustrated. In this model the charged massive vector multiplet of weak interactions contains besides $W^{\pm}$ gauge bosons and the Higgs scalars $H^{\pm}$, two spin-1/2 dirac fermions ω_1^- and ω_2^+ given by the linear combination of winos and higgsinos

$$\omega_1 = \sqrt{2} P_L \chi_{12} - i P_R \lambda^- \tag{1}$$

$$\omega_2 = \sqrt{2} P_L \chi_{21} - i P_R \lambda^+. \tag{2}$$

Here the Majorana fermions χ_{12}, χ_{21} and $\lambda^{\pm}$ are the supersymmetric partners of H^-, H^+ and $W^{\pm}$. If Susy is unbroken all these particles have a common mass m_W. Gauge invariance is broken by an Higgs sector composed by two Higgs doublets with opposite ipercharge ± 1, needed to give mass in a Susy invariant mode to both up and down quarks, and by an Higgs singlet N whose ipercharge is zero. Once Goldstone bosons are absorbed as the longitudinal degree of freedom of $W^{\pm}$ and Z^0 gauge bosons, we remain with 7 physical Higgs scalars: $H^{\pm}$, H^0, h_i^0 (i =1,2,3,4). $H^{\pm}$ and H^0 have the same mass as $W^{\pm}$ and Z^0 while h_1^0,...,h_4^0 have a common mass m_h. As far as the particle content is concerned, the proliferation of Higgs bosons is the only difference from the mere supersymmetrization of SM.

In a renormalizable theory of spin-$\frac{1}{2}$ and spin-1 particle, the tree level value of the gyromagnetic ratio is 2. This is strictly tied with tree level unitarity [3, 4]. However quantum effects can spoil this property. In particular, for vector multiplets, h could be nonzero due to loop effects.

Susy implies a strict relation between the following couplings

$$\frac{g}{m_W}\overline{\omega}_i\sigma_{\mu\nu}\omega_i F^{\mu\nu} \qquad\qquad g W^+_\mu W^-_\nu F^{\mu\nu}$$

$$\frac{g}{m_W}\epsilon^{\mu\nu\rho\sigma}(\partial_\mu H^+ W^-_\nu - \partial_\mu H^- W^+_\nu)F_{\rho\sigma}.$$

The most general CP and $U(1)_{e.m.}$ invariant $WW\gamma$ vertex when all particle are on mass shell is [5, 6]

$$M_{\mu\alpha\beta} = ie\{A[2p_\mu g_{\alpha\beta} + 4(q_\alpha g_{\beta\mu} - q_\beta g_{\mu\alpha})] + 2\Delta K_{WW}(q_\alpha g_{\beta\mu} - q_\beta g_{\mu\alpha}) + 4\frac{\Delta Q}{m_W^2}p_\mu q_\alpha q_\beta\} \quad (3)$$

where $p-q$, $p+q$, $2q$ are the momenta of the incoming and outgoing W^+ and of the incoming photon. In the standard model and in its supersymmetric version at tree level $A=1$, while for the anomalous magnetic dipole and electric quadrupole moments we have $\Delta K_{WW} = \Delta Q = 0$.

The charginos ω_1 and ω_2 whose electric charges are $e_{\omega_1} = -e$ and $e_{\omega_2} = +e$ $(e>0)$ can have an anomalous magnetic moment (in spite of chiral fermion $g_{1/2} = 2$ in exact Susy theories) $a_{\omega_i} = \frac{g_{\omega_i}-2}{2}$, given by the coefficient of

$$\frac{1}{2m_W}e_{\omega_i}\bar{\omega}_i\sigma^{\mu\nu}q_\nu\omega_i\varepsilon_\mu. \qquad (4)$$

with q and ε_μ the momentum and polarization vector of the incoming photon. The presence of this term is due to the fact we can embed the anomalous magnetic moment of gauge fermion in a supersymmetric invariant term [7, 2] while we can not do the same thing for chiral fermions [8].

Besides this we must consider the off-diagonal magnetic transition ΔK_{WH} between the spin-1 and spin-0 states in the vector multiplet. It is characterized, when all particles are on mass shell, as the coefficient of

$$\frac{e}{m_W}\varepsilon^{\mu\nu\rho\sigma}p_\rho q_\sigma\varepsilon_\mu\varepsilon'_\nu \qquad (5)$$

where $p, \varepsilon'_\nu, q, \varepsilon_\mu$ are the moment and polarization vector of the incoming W^+ and γ respectively. Supersymmetric sum rules foresee

$$\Delta K_{WW} = a_{\omega_1} = a_{\omega_2} = \Delta K_{WH}. \tag{6}$$

This forecast should be valid separately for the one loop contribution due to particles owing to every single mass supermultiplet unless we try to separate the contributions due to the exchange of quarks and leptons. This last statement is strictly tied with anomaly cancellation as proved in [2] .

We analyse the Susy magnetic moments sum rules for massless and massive ordinary fermions in sections 2 and 3 respectively and the Susy breaking case in section four. We report the whole set of function we found in this last case in Appendix. A comment on the size of the anomalous magnetic moment of W in the SM is given in the end of section 4 when we compare the SM with the exact and broken susy cases.

2 Susy sum rules: The massless fermions case

Bardeen et al. calculated ΔK_{WW} in the standard model [5, 9] and Bilchak et al. showed [2] in the case of massless ordinary fermions that ΔK_{WW} and a_{ω_i} have equal values. They verified this is true also if we consider separately the supermultiplets of photon, lepton, quarks, Z boson and Higgs and found

$$\begin{aligned}
\Delta K^{\gamma}_{WW} = \quad & a_{\omega_i} = \quad \frac{\alpha_{e.m.}}{\pi} & (7)\\
\Delta K^{l}_{WW} = \quad & a^{l}_{\omega_1} + \quad \frac{g^2}{32\pi^2} = a^{l}_{\omega_2} - \frac{g^2}{32\pi^2} = 0 & (8)\\
\Delta K^{q}_{WW} = \quad & a^{q}_{\omega_1} - \quad \frac{g^2}{32\pi^2} = a^{q}_{\omega_2} + \frac{g^2}{32\pi^2} = 0 & (9)\\
\Delta K^{Z}_{WW} = \quad & a^{Z}_{\omega_i} = \quad \frac{g^2}{16\pi^2}[\frac{4}{\rho} - \frac{1}{2} + \int_0^1 dx \frac{x^3 - 3x^2 + 4x - 4}{x^2 + \rho(1-x)}] & (10)\\
\Delta K^{H}_{WW} = \quad & a^{H}_{\omega_i} = \quad \frac{g^2}{16\pi^2}[\frac{1}{2} + \int_0^1 dx \frac{x^2(1-x)}{x^2 + \mu(1-x)}] & (11)
\end{aligned}$$

Here g is the weak coupling constant while $\rho = (\frac{m_Z}{m_W})^2$ and $\mu = (\frac{m_h}{m_W})^2$. We focalized our attention mainly on the contribution due to quarks, leptons and their superpartners.

The diagrams contributing to ΔK_{WW} for one generation of quarks and leptons are three, two with the up and down type quarks running in the loop and the photon leg attached to one of these, and only one for leptons because the neutrino is chargeless.

Three other diagrams have the same structure with the fermions substituted by squarks and sleptons. These graphs are illustrated in Fig.1-2.

As far as the contributions of the supermultiplets are considered, our results agree with the Bilchak et al. ones [2]. However we find some difference as far as the separate quark and squark contribution to ΔK_{WW} are concerned.

We find for one generation of massless fermions

$$\Delta K_{WW}(q) = \frac{-3g^2}{96\pi^2} \qquad \Delta K_{WW}(\tilde{q}) = \frac{3g^2}{96\pi^2} \tag{12}$$

$$\Delta K_{WW}(l) = \frac{-g^2}{96\pi^2} \qquad \Delta K_{WW}(\tilde{l}) = \frac{g^2}{96\pi^2}. \tag{13}$$

Indeed, if we call N_c the number of colours and $q_{u/d}$ the electromagnetic charges of the up and down quarks in units of e ($q_d = -\frac{1}{3}$, $q_u = \frac{2}{3}$), we have[2]

$$\Delta K_{WW}(q) = \frac{g^2}{96\pi^2} N_c(q_d - q_u) \tag{14}$$

This is a rather delicate point and therefore needs a careful analysis to explain the relative minus sign between charges in equation 14. As we can see in Fig.1, the graphs with the photon attached to the up and down quarks can be obtained one from the other with the substitutions $u \leftrightarrow d, \alpha \leftrightarrow \beta$ and $(p-q)_{in} \leftrightarrow (p+q)_{out}$. Last substitution is equivalent to $p \leftrightarrow -p$. From this observations and from equation 3 we deduce that if $M_a^{\mu\alpha\beta}(u,d)$ is the vertex contribution of graph a) with the u, d-dependence enclosed in the coefficients $A(u,d), \Delta K_{WW}(u,d), \Delta Q(u,d)$, the vertex contribution of graph b) is $M_b^{\mu\alpha\beta}(u,d) = -M_a^{\mu\alpha\beta}(d,u)$. This brings as a consequence that if the contributions of graph a) to ΔK_{WW} is $\frac{g^2}{96\pi^2} N_c q_d$ (equal to the electronic contribution), the sum with graph b) gives rise to equation 14.

Analogously the same graphs produce the following contribution to the electric quadrupole moment

$$\Delta Q(q) = \frac{-g^2}{72\pi^2} N_c(q_d - q_u). \tag{15}$$

This reasoning can be applied to the charge renormalization coefficient A and is valid for the sum of the squark loop graphs of Fig.2, too. For these scalar graphs we obtain results opposite to 14 , 15 .

Nevertheless if we consider the anomalous term $N^{\mu\alpha\beta} = B\varepsilon^{\mu\alpha\beta\nu} p_\nu$ that can be generated by fermion loops, it receives from graphs a) and b) of Fig.1 contributions $B_b(u,d) =$

[2]I wish to thank Prof. A. Van Proeyen for useful discussion on this point.

$B_a(d,u)$ without any change of sign because $\varepsilon^{\mu\alpha\beta\nu}p_\nu$ is invariant under the previous set of substitutions. Therefore, considering one complete fermion generation we have

$$B(q,l) = [N_c(q_d + q_u) + q_e] \cdot cost = 0, \tag{16}$$

so there is no problem for anomaly cancellation.

This point has been previously overlooked and, hence, our computation leads to a different prediction for ΔK_{WW} in the standard model even if it does not affect ΔK_{WW} in the massless fermions MSSM, because quark and squark contributions cancel each other anyway. Indeed, taking into account the presence in SM of the three fermionic generations, the final quarks and leptons exchange contribution is a sizeable one.

If we consider massless ordinary fermions, the chargino ω_1 is only coupled to [10] $d\tilde{u}_L$ and $e\tilde{\nu}_L$ and not to $u\tilde{d}_L$ and $\nu\tilde{e}_L$, with the opposite assignation for ω_2. These couplings give rise to the anomalous magnetic moment one loop corrections illustrated in Fig.3-4. Here we list every single contribution putting in evidence the particles whose propagators we met in the loops.

$$\begin{array}{rcl}
a_{\omega_1}(d\tilde{u}\tilde{u}) = \dfrac{2g^2}{32\pi^2} & \qquad & a_{\omega_2}(u\tilde{d}\tilde{d}) = \dfrac{g^2}{32\pi^2} \\
a_{\omega_1}(dd\tilde{u}) = \dfrac{-g^2}{32\pi^2} & \qquad & a_{\omega_2}(uu\tilde{d}) = \dfrac{-2g^2}{32\pi^2} \\
a_{\omega_1}(ee\tilde{\nu}) = \dfrac{-g^2}{32\pi^2} & \qquad & a_{\omega_2}(\nu\tilde{e}\tilde{e}) = \dfrac{g^2}{32\pi^2}.
\end{array}$$

Our results agree with [2] .

In the massless fermions case we are considering, the vertex $WH\gamma$ is one loop affected only by sleptons and squarks (coupling $\tilde{u}_L\tilde{d}_L H = \frac{-igm_W}{\sqrt{2}}$ and analogues) since leptons and quarks have vanishing Yukawa couplings [10]. Besides this a scalar loop cannot provide terms like $\varepsilon^{\mu\nu\rho\sigma}p_\rho q_\sigma \varepsilon_\mu \varepsilon'_\nu$, so we have

$$\Delta K_{WH}^{q,\tilde{q},l,\tilde{l}} = 0. \tag{17}$$

This completes the verification of the sum rules for the quark and lepton multiplets.

We list here the ΔK_{WH} contributions due to the γ, Z, H supermultiplets illustrated in Fig.6, so, taking into account eqations 7, 10, 11, the verification is complete for the other multiplets, too.

$$\Delta K_{WH}(\tilde{\gamma}\omega_i\omega_i) = \ \tfrac{1}{2}\Delta K_{WH}^{\gamma} = \ \frac{\alpha_{e.m.}}{2\pi} \tag{18}$$

$$\Delta K_{WH}(\zeta\omega_i\omega_i) = \tfrac{1}{2}\Delta K^Z_{WH} = \frac{g^2}{32\pi^2}[\frac{4}{\rho} - \frac{1}{2} + \int_0^1 dx \frac{x^3 - 3x^2 + 4x - 4}{x^2 + \rho(1-x)}] \qquad (19)$$

$$\Delta K_{WH}(\tilde{h}\omega_i\omega_i) = \tfrac{1}{2}\Delta K^H_{WH} = \frac{g^2}{32\pi^2}[\frac{1}{2} + \int_0^1 dx \frac{x^2(1-x)}{x^2 + \mu(1-x)}] \qquad (20)$$

Here $\tilde{\gamma}, \zeta$ and $\tilde{h}$ are the fermionic partner of γ, Z and h_i respectively.

3 Susy sum rules: Massive fermions

We can rely upon the results of the previous section as far as the first two fermions generations are concerned, because the fermions masses involved can be neglected compared to m_W. Nevertheless in a realistic model we must consider the fact that the top quark must be heavier than W, too.

We shall consider two cases: a) m_W negligible with respect to m_t and fixed ratio $(\frac{m_b}{m_t})^2 = r$; b) fixed ratio $(\frac{m_W}{m_t})^2 = \alpha$ with $m_b = 0$. Given the established hierarchy $m_b << m_W < m_t$ (with m_t of the order of twice m_W according to the precision tests of the standard model physics at LEP (see for example [11]) and the first indication for direct top evidence at Tevatron [12]), the approximation of case b) in which terms of $O(\frac{m_b}{m_t})$ are neglected is certainly more realistic. In both cases we'll consider massless leptons.

Considering massive ordinary fermions brings as a consequence a multiplication of the couplings appearing in the lagrangian. In particular we have new vertices like $\bar{t}bH$ and the fermion number violating $\omega_1^c\bar{t}\tilde{b}_R$ and $\omega_2^c\bar{b}\tilde{t}_R$ [13]. These new couplings generate the diagrams illustrated in Fig.5-7-8 which contribute to ΔK_{WH} and a_{ω_i}.

Case a) was just analyzed in [14] and we report here only the final result valid for the contributions of the quarks and leptons multiplets

$$\Delta K^{ql}_{WW} = a^{ql}_{\omega_1} = a^{ql}_{\omega_2} = \Delta K^{ql}_{WH} = \frac{-g^2}{32\pi^2}F(r) \qquad (21)$$

with

$$F(r) = \frac{1}{(1-r)^3}[r^3 + 11r^2 - 13r + 1 - 4r(1+2r)\ln r]. \qquad (22)$$

Here the result for the third generation coincides with the three generations one. This time we have an universal non null function $F(r)$ with which we can express the whole set of anomalous magnetic moments. In the interesting cases $m_b = 0$, $m_t = 0$, $m_b = m_t$ we have respectively $F(0) = 1$, $F(\infty) = -1$ and $F(1) = 1$.

For case b) we report the whole set of functions we obtain. In particular we find it relevant that our results restricted to the SM case exhibit some difference with the values previously reported in the literature. If we put $a = (\frac{m_t}{m_W})^2$ and $b = (\frac{m_b}{m_W})^2$ we obtain

$$\Delta K_{WW}(bbt) = \frac{g^2 N_c q_b}{32\pi^2}\int_0^1 dx \frac{x^4 + x^3(a-b-1) + x^2(2b-a)}{bx + a(1-x) - x(1-x)} \qquad (23)$$

$$\Delta K_{WW}(\tilde{b}\tilde{b}\tilde{t}) \quad = \quad \frac{-g^2 N_c q_b}{16\pi^2}\int_0^1 dx \frac{(x^3-x^2)(b-a-1+2x)}{bx+a(1-x)-x(1-x)} \tag{24}$$

$$a_{\omega_1}(b\tilde{t}\tilde{t}) \quad = \quad \frac{g^2 N_c q_t}{16\pi^2}\int_0^1 dx \frac{x(x-1)[b(x-2)+x]}{ax+b(1-x)-x(1-x)} \tag{25}$$

$$a_{\omega_1}(bb\tilde{t}) \quad = \quad \frac{g^2 N_c q_b}{16\pi^2}\int_0^1 dx \frac{x^2[b(x+1)+x-1]}{bx+a(1-x)-x(1-x)} \tag{26}$$

$$a_{\omega_1}(t\tilde{b}\tilde{b}) \quad = \quad \frac{g^2 N_c q_b}{16\pi^2}\int_0^1 dx \frac{x^2 b(1-x)}{bx+a(1-x)-x(1-x)} \tag{27}$$

$$a_{\omega_1}(tt\tilde{b}) \quad = \quad \frac{g^2 N_c q_t}{16\pi^2}\int_0^1 dx \frac{x^2 b(1-x)}{ax+b(1-x)-x(1-x)} \tag{28}$$

$$\Delta K_{WH}(tbb) = \frac{-g^2 N_c q_b}{16\pi^2}\int_0^1 dx \frac{x[a(1-x)-bx]}{bx+a(1-x)-x(1-x)}. \tag{29}$$

We obtain the corresponding contributions for $\Delta K_{WW}(btt)$, $\Delta K_{WW}(\tilde{b}\tilde{t}\tilde{t})$, $a_{\omega_2}(t\tilde{b}\tilde{b})$, $a_{\omega_2}(tt\tilde{b})$, $a_{\omega_2}(b\tilde{t}\tilde{t})$, $a_{\omega_2}(bb\tilde{t})$, $\Delta K_{WH}(btt)$, with the substitutions $q_b \leftrightarrow -q_t$, $m_b \leftrightarrow m_t$.

The relative minus sign between the charges q_b and q_t has already been explained in the case of ΔK_{WW}. As far as a_{ω_i} is concerned it is simply due to a_{ω_i} definition (4) which contains e_{ω_i}. Writing explicitly the matrix elements for the two ΔK_{WH} quark contributions depicted in Fig. 5 and using usual trace properties, the same substitution statement is easily verified.

Summing up the whole set of results 23, ..., 29 and taking into account a massless lepton generation we obtain

$$\Delta K_{WW}^{q\tilde{q}l\tilde{l}} = a_{\omega_1}^{q\tilde{q}l\tilde{l}} = a_{\omega_2}^{q\tilde{q}l\tilde{l}} = \Delta K_{WH}^{q\tilde{q}l\tilde{l}} = \frac{g^2}{16\pi^2}\int_0^1 dx \frac{(1-3x)(ax-b(1-x))}{ax+b(1-x)-x(1-x)} \tag{30}$$

Inserting $b=0$ and expressing everything as a function of $\alpha = (\frac{m_W}{m_t})^2$ we find

$$\Delta K_{WW}^{q\tilde{q}l\tilde{l}} = a_{\omega_1}^{q\tilde{q}l\tilde{l}} = a_{\omega_2}^{q\tilde{q}l\tilde{l}} = \Delta K_{WH}^{q\tilde{q}l\tilde{l}} = \frac{-g^2}{32\pi^2} G(\alpha) \tag{31}$$

with

$$G(\alpha) = \frac{2}{\alpha^2}[3\alpha + (3-2\alpha)\ln(1-\alpha)]. \tag{32}$$

This function has $\lim_{\alpha\to 0} G(\alpha) = 1$ so reproducing the case of negligible m_W. Again the anomalous magnetic moments supersymmetric sum rules are exactly verified.

The complete expression obtained using the realistic value $m_b = 5GeV$ give results which differ from those obtained with $G(\alpha)$ by O(10%) if $m_t \simeq 100GeV$ and only by O(1%) if $m_t > 160GeV$.

The function $G(\alpha)$ exhibit a divergence for $m_t = m_W$ because in this and in the more general case $m_b + m_t = m_W$ the diagrams we studied have a singularity in the physical region due to the presence of the threshold for the $W \to tb$ decay. Our calculation should be reliable provided m_t differs from m_W more than the W decay width.

If we limit ourselves to the SM contribution to ΔK^{ql}_{WW} due to the third generation, keeping m_t as the only non null fermion mass we have

$$\Delta K^{ql}_{WW}(SM) = \frac{-g^2}{96\pi^2}\frac{1}{\alpha^3}[4\alpha^3 - 3\alpha^2 + 18\alpha + 6(3-2\alpha)\ln(1-\alpha)]. \quad (33)$$

This result differs from those found with the same assumptions in [15] and [16]. Such difference can be traced back to the erroneous summation of the two quarks diagrams analogously to what happens in the massless case [5, 2] we clarified before.

We note that equation 33 reduce to $\Delta K^{ql}_{WW}(SM) = \frac{-g^2}{24\pi}$ if α goes to 0 or ∞ and this is the same result we obtained in the massless fermion case . As a further ("a posteriori") check of validity of 33 we observe that summing up with the supersymmetric contributions we find the same universal function $G(\alpha)$ that arises independently in the computation of a_{ω_1}, a_{ω_2} and ΔK_{WH}.

4 Soft breaking with scalar masses

As a final step we calculated the total contribution to the four quantities we considered in the MSSM with Susy broken explicitly but softly by an universal mass $\tilde{m}$ for every scalar particle we have in the theory, to make a comparison with the unbroken case and search for a possible new rule relating magnetic moments in broken Susy multiplets.

This choice of Susy breaking is not only the simplest but can also be seen as one of the possible low energy remnant of string theory. Particularly in a large class of string scenarios (symmetric orbifolds) scalar masses should be largely bigger than gaugino masses and in spite of a general lack of universality they should be nearly universal because of the weak dependence from their corresponding modular weights [17].

The tree level $\tilde{m}$ introduction effect is only a shift in scalar mass eigenstates which does not affect directly any other coupling including $\omega_i q_j \tilde{q}_k$, $q_i q_j H$ and $\tilde{q}_i \tilde{q}_j H$ which depend only on fermion masses generated through their Yukawa couplings.

The whole set of results we obtained is reported in Appendix where the dependence from the new parameter $\delta = (\frac{m_W}{\tilde{m}})^2$ is put in evidence. These results are not very enlightening and so we examined the quark and lepton multiplet contribution for 1 fermion generation in three particular cases: i) massless fermions; ii) only one massive fermion (top); iii) one heavy completely isomassive fermions generation.

In case i) the values reported in equations 8 , 9 and 17 for the unbroken Susy case get modified when $\tilde{m} >> m_W$:

$$\Delta K_{WW}^{q\tilde{q}l\tilde{l}} = \frac{-g^2}{24\pi^2} \tag{34}$$

$$a_{\omega_1}^{q\tilde{q}l\tilde{l}} = a_{\omega_2}^{q\tilde{q}l\tilde{l}} = \frac{g^2}{48\pi^2} \tag{35}$$

$$\Delta K_{WH}^{q\tilde{q}l\tilde{l}} = 0. \tag{36}$$

In case ii) the corresponding starting value is $\frac{-g^2}{32\pi^2}$ (see equation 21 in the $r \to 0$ limit), while for $\tilde{m} >> m_{t/W}$ the results are

$$\Delta K_{WW}^{q\tilde{q}l\tilde{l}} = \frac{-g^2}{24\pi^2} \tag{37}$$

$$a_{\omega_1}^{q\tilde{q}l\tilde{l}} = a_{\omega_2}^{q\tilde{q}l\tilde{l}} = \frac{g^2}{48\pi^2} \tag{38}$$

$$\Delta K_{WH}^{q\tilde{q}l\tilde{l}} = \frac{-g^2}{32\pi^2}. \tag{39}$$

If we change the mass hierarchy setting $m_t >> \tilde{m} >> m_W$ they become

$$\Delta K_{WW}^{q\tilde{q}l\tilde{l}} = a_{\omega_2}^{q\tilde{q}l\tilde{l}} = \frac{-g^2}{24\pi^2} \tag{40}$$

$$a_{\omega_1}^{q\tilde{q}l\tilde{l}} = \frac{-g^2}{96\pi^2} \tag{41}$$

$$\Delta K_{WH}^{q\tilde{q}l\tilde{l}} = \frac{-g^2}{32\pi^2}. \tag{42}$$

In the final case of a completely isomassive generation we start from $\frac{-g^2}{24\pi^2}$ and in the case of $\tilde{m} >> m$ with m the common fermion mass we have

$$\Delta K_{WW}^{q\tilde{q}l\tilde{l}} = \Delta K_{WH}^{q\tilde{q}l\tilde{l}} = \frac{-g^2}{24\pi^2} \tag{43}$$

$$a_{\omega_1}^{q\tilde{q}l\tilde{l}} = a_{\omega_2}^{q\tilde{q}l\tilde{l}} = \frac{g^2}{48\pi^2}. \tag{44}$$

Instead of these if we set $m >> \tilde{m} >> m_W$ we obtain

$$\Delta K_{WW}^{q\tilde{q}l\tilde{l}} = \Delta K_{WH}^{q\tilde{q}l\tilde{l}} = a_{\omega_1}^{q\tilde{q}l\tilde{l}} = a_{\omega_2}^{q\tilde{q}l\tilde{l}} = \frac{-g^2}{24\pi^2}. \tag{45}$$

As we can see from these results we haven't a common clear sign relating Susy breaking with the new sum rules we could write in every single case. The problem is even more complicated if we consider non negligible m_t or m_W compared to $\tilde{m}$, in which case the four magnetic moments we considered have four different and apparently uncorrelated values. Indeed this happens with an explicit Susy breaking but we have a first indication things can hardly go better in the case of spontaneous breaking because quark and lepton supermultiplet contributions should always be present and they are rather independent from the contributions of the other multiplets (the fermion mass hierarchy is not fixed by Susy).

We conclude our work with some tables giving the total results for ΔK_{WW} considering the whole set of diagrams contributing to it, γ, Z and H multiplets included, in the case of the SM (Table 1), of its minimal exactly supersymmetric version (Table 2) and in the case of Susy broken by the universal scalar mass $\tilde{m}$.

In the case of SM and its minimal exactly supersymmetric version we give the results in function of $\mu = (\frac{m_h}{m_W})^2$ and $\alpha = (\frac{m_W}{m_t})^2$. In the case of broken Susy they are expressed in function of μ and $\delta = (\frac{m_W}{\tilde{m}})^2$ at fixed values of α belonging to the favourite range 100-200GeV for m_t.

Throughout these computations we used $\sin^2\theta_W = .2325$ and $\rho = (\frac{m_Z}{m_W})^2 = 1.29$.

The standard model results differs significantly from the published one because of the non cancellation of the fermion contributions (three generations). The most striking result is that ΔK_{WW} is negative, unless a light Higgs is present, and rather small. This result contrasts with what was previously found in the literature. As we can see in Table 1, ΔK_{WW} is a decreasing function both of m_t and m_h. If we assume the CDF result [12] $m_t = 174 \pm 17$ GeV and use the lower bound $m_t = 157$ GeV we find the largest m_h that allows a positive ΔK_{WW}. We find

$$\Delta K_{WW} > 0 \Longrightarrow m_h \leq 81 GeV \tag{46}$$

It must be remembered the LEP1 limit [18] $m_h > 63.8$ GeV that can be raised to $80 - 90$ GeV at LEP2.

Unfortunately the corrections to $g_W = 2$ remains always of the order 10^{-3}, both in the standard and in the supersymmetric case, making impossible an experimental verification as today we know only g_W is $O(1)$ and in the near future we will probably know it at best with a precision ten times better [19].

The results reported in Table 3 should be taken as an indication of the departure from the unbroken Susy case in the simplest possible approach where the entire Susy breaking is accounted for by a universal scalar mass. Clearly a more detailed analysis should take into account the influence of a specific breaking of supergravity at a large scale down to low energy through the evolution of the full Susy parameter spectrum.

A final comment on the size of the contributions in the tables is in order. It's well known that the supersymmetric corrections to the $g - 2$ of the muon in MSSM is particularly small [10, and references therein] and, indeed, it does not lead to significant constraints on the sparticle masses given the present experimental lower bounds on them. On the

contrary the supersymmetric contributions to the $g-2$ of the W boson is of the same order of magnitude as the SM corrections. This is due to the fact that the ratio of the W and Susy masses is much larger than the analogous ratio of the muon to the Susy masses.

As we can see from table 2, exact Susy predict a rather different range of values compared with the SM one, resulting in a probable difference of sign between them. However the explicit supersymmetry breaking with $\tilde{m} > m_W$ (phenomenology states Susy breaking cannot be to much little) modifies in a sizeable way the squarks and sleptons contribution and the H^+H^0, $W^+h_1^0$, $H^+h_2^0$ ones depicted in Fig.9 [2]. This leads to foresee results nearer to the SM and with the same sign.

Acknowledgment

I wish to thank Antonio Masiero for his guide and patient support, Antoine Van Proeyen for his kind interest and Ferruccio Feruglio for useful discussions.

Appendix

We report the whole set of scalar contributions to ΔK_{WW} modified by the introduction of the universal Susy breaking parameter $\tilde{m}$ as a function of $a_i = (\frac{m_{u_i}}{m_W})^2$, $b_i = (\frac{m_{d_i}}{m_W})^2$ and $\delta = (\frac{m_W}{\tilde{m}})^2$ with $u_i = u, c, t$ and $d_i = d, s, b$

$$
\begin{aligned}
\Delta K_{WW}(\tilde{d}_i\tilde{d}_i\tilde{u}_i) &= \frac{g^2 N_c q_{d_i}}{16\pi^2}\int_0^1 dx \frac{\delta(x^2-x^3)(b_i - a_i - 1 + 2x)}{b_i x + a_i(1-x) + \frac{1}{\delta} - x(1-x)} \\
\Delta K_{WW}(\tilde{d}_i\tilde{u}_i\tilde{u}_i) &= \frac{-g^2 N_c q_{u_i}}{16\pi^2}\int_0^1 dx \frac{\delta(x^2-x^3)(a_i - b_i - 1 + 2x)}{a_i x + b_i(i-x) + \frac{1}{\delta} - x(1-x)}.
\end{aligned}
$$

Here and in the following, leptonic contributions can be easily obtained from those which contains q_{d_i} with the substitutions $N_c q_{d_i} \to q_{l_i}$, $a_i \to a'_i$ and $b_i \to b'_i$ with $a'_i = (\frac{m_{\nu_i}}{m_W})^2$ and $b'_i = (\frac{m_{l_i}}{m_W})^2$.

Considering quark t as the only massive fermion and setting $\alpha = (\frac{m_W}{m_t})^2$, the scalar quarks and leptons give

$$
\begin{aligned}
\Delta K_{WW}(\tilde{d}\tilde{d}\tilde{u}) &= K_{WW}(\tilde{s}\tilde{s}\tilde{c}) = K_{WW}(\tilde{e}\tilde{e}\tilde{\nu}) = K_{WW}(\tilde{\mu}\tilde{\mu}\tilde{\nu}) = K_{WW}(\tilde{\tau}\tilde{\tau}\tilde{\nu}) \\
&= \frac{-g^2}{16\pi^2}\int_0^1 dx \frac{\delta(x^2-x^3)(2x-1)}{1-\delta x(1-x)} \\
\Delta K_{WW}(\tilde{d}\tilde{u}\tilde{u}) &= K_{WW}(\tilde{s}\tilde{c}\tilde{c}) = 2K_{WW}(\tilde{d}\tilde{d}\tilde{u}) \\
\Delta K_{WW}(\tilde{b}\tilde{b}\tilde{t}) &= \frac{-g^2}{16\pi^2}\int_0^1 dx \frac{\delta(x^2-x^3)(\alpha(2x-1)-1)}{\alpha + \delta(1-x) - \alpha\delta x(1-x)} \\
\Delta K_{WW}(\tilde{b}\tilde{t}\tilde{t}) &= \frac{-2g^2}{16\pi^2}\int_0^1 dx \frac{\delta(x^2-x^3)(\alpha(2x-1)+1)}{\alpha + \delta x - \alpha\delta x(1-x)}.
\end{aligned}
$$

The ordinary Higgs contributions get modified by $\tilde{m}$ introduction, too. The functions reported in [2] become

$$
\begin{aligned}
\Delta K_{WW}(H^+H^+H^0) &= \frac{g^2}{16\pi^2}[\frac{1}{6} + \frac{1}{2}\int_0^1 dx \frac{x^2(x^2-2x-\frac{1}{\delta})}{x^2 + \rho(1-x) + \frac{1}{\delta}}] \\
\Delta K_{WW}(W^+W^+h_1^0) &= \frac{g^2}{16\pi^2}[\frac{1}{6} + \frac{1}{2}\int_0^1 dx \frac{x^2(x^2-2x+4)}{x^2 + (\mu + \frac{1}{\delta})(1-x)}] \\
\Delta K_{WW}(H^+H^+h_2^0) &= \frac{g^2}{16\pi^2}[\frac{1}{6} + \frac{1}{2}\int_0^1 dx \frac{x^2(x^2-2x-\frac{1}{\delta})}{x^2 + \mu(1-x) + \frac{1}{\delta}}],
\end{aligned}
$$

where we used $\rho = (\frac{m_Z}{m_W})^2$ and $\mu = (\frac{m_h}{m_W})^2$. We recover exactly supersymmetric functions simply by letting δ goes to infinity. The same is true for the modified contributions to a_{ω_i} we list here for arbitrary quark mass.

$$
\begin{aligned}
a_{\omega_1}(d_i\tilde{u}_i\tilde{u}_i) &= \frac{g^2 N_c q_{u_i}}{16\pi^2}\int_0^1 dx \frac{x(x-1)[b_i(x-2)+x]}{(a_i+\frac{1}{\delta})x + b_i(1-x) - x(1-x)} \\
a_{\omega_1}(d_i d_i\tilde{u}_i) &= \frac{g^2 N_c q_{d_i}}{16\pi^2}\int_0^1 dx \frac{x^2[b_i(x+1)+x-1]}{b_i x + (a_i+\frac{1}{\delta})(1-x) - x(1-x)} \\
a_{\omega_1}(u_i\tilde{d}_i\tilde{d}_i) &= \frac{g^2 N_c q_{d_i}}{16\pi^2}\int_0^1 dx \frac{x^2(b_i+\frac{1}{\delta})(1-x)}{(b_i+\frac{1}{\delta})x + a_i(1-x) - x(1-x)} \\
a_{\omega_1}(u_i u_i\tilde{d}_i) &= \frac{g^2 N_c q_{u_i}}{16\pi^2}\int_0^1 dx \frac{x^2(b_i+\frac{1}{\delta})(1-x)}{a_i x + (b_i+\frac{1}{\delta})(1-x) - x(1-x)}
\end{aligned}
$$

As in the exactly supersymmetric case we obtain the corresponding a_{ω_2} contributions with the substitutions $q_{d_i} \leftrightarrow -q_{u_i}$, $a_i \leftrightarrow b_i$.

We have not new contribution to ΔK_{WH} due to the introduction of the universal scalar mass because the off diagonal magnetic transition does not receive any influence from scalar loops.

References

[1] S. Ferrara and M. Porrati, Phys. Lett. B288 (1992) 85;

[2] Bilchak, R. Gastmans and A. van Proeyen, Nucl. Phys. B273 (1986) 46;

[3] S. Weinberg, in Lectures on Elementary Particles and Quantum Field Theory, Proc. of the Summer Institute, Brandeis University, 1970, ed. S. Deser (MIT Press, Cambridge, MA, 1970), Vol. I;

[4] S. Ferrara, M. Porrati and V. Telegdi, Phys. Rev. D46 (1992) 3529;

[5] W. A. Bardeen, R. Gastmans and B. Lautrup, Nucl. phys. B46 (1972) 319;

[6] K. Hagiwara, R. D. Peccei, D. Zeppenfeld and K. Hikasa, Nucl. Phys. B282 (1987) 253;

[7] R. W. Robinett, Phys. Rev. D31 (1985) 1657;

[8] S. Ferrara and E. Remiddi, Phys. Lett. B53 (1974) 347;

[9] K. O. Mikaelian, M. A. Samuel and D. Brown, Lett. Nuovo Cim. 27 (1980) 211;

[10] H. E. Haber and G. L. Kane, Phys. Rep. 117 (1985) 75;

[11] The LEP Collaborations: ALEPH, DELPHI, L3, OPAL, Phys. Lett. B276 (1992) 247;

[12] F. Abe et al., The CDF Collaboration, FERMILAB-PUB-94/097-E submitted to Phys. Rev. D; FERMILAB-PUB-94/116-E submitted to Phys. Rev. Lett.;

[13] J. F. Gunion and H. E. Haber, Nucl. Phys. B272 (1986) 1;

[14] S. Ferrara and A. Masiero, CERN TH-6846/93, Proc. of "SUSY 93" Northeastern University, Boston, USA (1993) and Proc of 26th Workshop, Eloisatron Project, "From Superstring to Supergravity", Erice, 1992;

[15] G. Couture and J. N. Ng, Z. Phys. C - Particles and Fields 35 (1987) 65;

[16] K. B. Samuel, M. A. Samuel and G. Li, Osu Research Note 218, May, 1989;

[17] L. E. Ibáñez and D. Lüst, Nucl. Phys. B382 (1992) 305; B. de Carlos, J. A. Casas and C. Muñoz, phys. Lett. B299 (1993) 234; B. de Carlos, J. A. Casas and C. Muñoz, CERN-TH 6922/93;

[18] J. Schwidling, Proc. of the Int. Europhysics Conference, Marseille, 1993;

[19] D. Zeppenfeld, Phys. Lett. B183 (1987) 380; U. Baur and D. Zeppenfeld, Nucl. Phys. B325 (1989) 253; E. N. Argyres et al., Phys. Lett. B272 (1991) 431; E. N. Argyres et al., Phys. Lett. B280 (1992) 324; M. A. Samuel, G. Li, N. Sinha, R. Sinha and M. K. Sundaresan, Phys. Lett. B280 (1992) 124;

$\alpha \rightarrow$ / $\mu \downarrow$	.64	.44	.25	.16	10^{-4}
10^{-2}	2.35	2.13	2.02	1.99	1.95
10^{-1}	1.53	1.30	1.20	1.17	1.13
.5	.70	.47	.37	.34	.30
1	.33	.11	2×10^{-3}	-.03	-.07
2	-.02	-.25	-.35	-.38	-.42
3	-.21	-.43	-.53	-.57	-.61
4	-.33	-.55	-.66	-.69	-.73
5	-.42	-.64	-.75	-.78	-.82
10	-.66	-.89	-.99	-1.02	-1.06
20	-.84	-1.07	-1.17	-1.20	-1.24
50	-1.01	-1.23	-1.33	-1.37	-1.41
100	-1.08	-1.31	-1.41	-1.44	-1.48

TABLE 1. $\Delta K_{WW} \times 10^{-3}$ in the Standard Model. $\mu = (\frac{m_h}{m_W})^2$, $\alpha = (\frac{m_W}{m_t})^2$. m_h is the mass of the only physical Higgs scalar in SM.

$\alpha \rightarrow$ / $\mu \downarrow$	.64	.44	.25	.16	10^{-4}
10^{-2}	3.06	2.89	2.83	2.81	2.81
10^{-1}	2.73	2.56	2.50	2.49	2.48
.5	2.46	2.28	2.22	2.21	2.21
1	2.36	2.18	2.12	2.11	2.10
2	2.28	2.10	2.04	2.03	2.02
3	2.24	2.06	2.00	1.99	1.98
4	2.21	2.03	1.97	1.96	1.96
5	2.20	2.02	1.96	1.95	1.94
10	2.16	1.98	1.92	1.91	1.90
20	2.13	1.95	1.89	1.88	1.87
50	2.11	1.93	1.87	1.86	1.86
100	2.11	1.93	1.87	1.86	1.85

TABLE 2. ΔK_{WW} in the minimal exactly supersymmetric version of the standard model. Here m_h in $\mu = (\frac{m_h}{m_W})^2$ is the mass of the neutral Higgs bosons which are not in the Z supermultiplet [2].

$\delta \rightarrow$		1			.25			.04	
$\mu \downarrow \alpha \rightarrow$	.64	.25	.16	.64	.25	.16	.64	.25	.16
10^{-2}	-2.93	-3.26	-3.27	-3.06	-3.44	-3.49	-3.44	-3.79	-3.83
10^{-1}	-2.59	-2.91	-2.93	-2.68	-3.06	-3.11	-3.06	-3.40	-3.45
.5	-2.32	-2.65	-2.66	-2.32	-2.70	-2.75	-2.67	-3.01	-3.06
1	-2.25	-2.57	-2.59	-2.17	-2.54	-2.60	-2.49	-2.83	-2.88
2	-2.21	-2.54	-2.55	-2.04	-2.42	-2.47	-2.31	-2.66	-2.71
3	-2.20	-2.53	-2.55	-1.98	-2.36	-2.41	-2.22	-2.57	-2.61
4	-2.20	-2.53	-2.54	-1.95	-2.32	-2.37	-2.16	-2.50	-2.55
5	-2.20	-2.53	-2.54	-1.92	-2.30	-2.35	-2.11	-2.46	-2.50
10	-2.20	-2.53	-2.54	-1.87	-2.25	-2.30	-1.99	-2.34	-2.38
20	-2.20	-2.53	-2.54	-1.83	-2.21	-2.26	-1.90	-2.25	-2.29
50	-2.20	-2.52	-2.54	-1.79	-2.17	-2.22	-1.82	-2.16	-2.21
100	-2.19	-2.51	-2.53	-1.77	-2.14	-2.20	-1.76	-2.10	-2.15

$\delta \rightarrow$		.01			.0001	
$\mu \downarrow \alpha \rightarrow$	.64	.25	.16	.64	.25	.16
10^{-2}	-3.61	-3.94	-3.97	-3.72	-4.04	-4.08
10^{-1}	-3.22	-3.56	-3.59	-3.33	-3.66	-3.69
.5	-2.83	-3.16	-3.20	-2.94	-3.27	-3.30
1	-2.65	-2.98	-3.02	-2.76	-3.09	-3.12
2	-2.47	-2.81	-2.84	-2.58	-2.91	-2.94
3	-2.38	-2.71	-2.74	-2.49	-2.81	-2.85
4	-2.31	-2.64	-2.68	-2.42	-2.75	-2.78
5	-2.26	-2.60	-2.63	-2.37	-2.70	-2.73
10	-2.13	-2.46	-2.50	-2.24	-2.57	-2.60
20	-2.03	-2.36	-2.40	-2.14	-2.47	-2.50
50	-1.93	-2.26	-2.29	-2.05	-2.38	-2.41
100	-1.87	-2.20	-2.23	-2.00	-2.33	-2.36

TABLE 3. $\Delta K_{WW} \times 10^{-3}$ in the MSSM with Susy broken by an universal scalar mass $\tilde{m}$. $\delta = (\frac{m_W}{\tilde{m}})^2$.